Formal Definition of a Pushdown Automata

A pushdown automaton is a 6-tuple \((Q, \Sigma, \Gamma, \delta, q_0, F)\), where \(Q, \Sigma, \Gamma,\) and \(F\) are all finite sets, and

1. \(Q\) is the set of states,
2. \(\Sigma\) is the input alphabet,
3. \(\Gamma\) is the stack alphabet,
4. \(\delta: Q \times \Sigma \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma)\) is the transition function,
5. \(q_0 \in Q\) is the start state, and
6. \(F \subseteq Q\) is the set of accept states.
PDA $M_1$ that recognizes $\{0^n1^n \mid n \geq 0\}$

Let $M_1$ be $(Q, \Sigma, \Gamma, \delta, q_1, F)$, where

$Q = \{q_1, q_2, q_3, q_4\}$,

$\Sigma = \{0, 1\}$,

$\Gamma = \{0, \$\}$,

$F = \{q_1, q_4\}$, and

$\delta$ is given by the following table, wherein blank entries signify $\emptyset$.

<table>
<thead>
<tr>
<th>Input: Stack:</th>
<th>0</th>
<th>$\varepsilon$</th>
<th>1</th>
<th>0</th>
<th>$\varepsilon$</th>
<th>0</th>
<th>$\varepsilon$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td>0</td>
<td>$\varepsilon$</td>
<td>0</td>
<td>0</td>
<td>$\varepsilon$</td>
<td>0</td>
<td>0</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>q2</td>
<td>${(q_2, 0)}$</td>
<td>${(q_3, \varepsilon)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q3</td>
<td>${(q_3, \varepsilon)}$</td>
<td></td>
<td></td>
<td>${(q_4, \varepsilon)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

{(q_2, $\$$)}