Free Throw Percentage Problem

I missed my first free-throw of the year. Don’t worry; I ended the season shooting 95% from the line (yeah, I’m pretty good).

My question to you is the following:

Was there necessarily a point during the season (mid game, perhaps), when I was shooting EXACTLY 90% from the line?

Solution using inequalities

Assuming this were possible, and letting \( m = \) free throws made and \( a = \) free throws attempted, there would have to be an \( m \) and \( a \) so that \( \frac{m}{a} < 90\% \) and

\[
\frac{m+1}{a+1} > 90\%
\]

so \( \frac{m}{a} < \frac{q}{10} \) and \( \frac{m+1}{a+1} > \frac{q}{10} \)

Combining these inequalities, we have \( 10m < 9a < 10m+1 \). But this is impossible, as \( 10m \) and \( 10m+1 \) are consecutive integers, so the integer \( 9a \) cannot be strictly between them.

Thus it is not possible to “jump over” 90%, so at some point I was shooting exactly 90%.